Uncertainties of Al Ain Geometrical Geoid

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Abstract:
An exercise of establishing a local geoid for the Eastern Region of Abu Dhabi Emirate has been attempted in three steps; the first attempt was by applying a geometrical geoid model using observed ellipsoidal heights. Secondly the EGM96 model was compared against the geoid undulations as derived from the geometrical geoid model from actual survey data. The third attempt was by the use of the resulting bias to best fit the EGM96 model to the local conditions; in an attempt to improve the results, hence transforming the bias corrected EGM96 to a best-fit local geoid.
Using the orthometric heights of the first order levelling network, Geoid heights in the Eastern region of Abu Dhabi Emirate has been determined with uncertainties that are typically better than 3cm. By the aid of the geometrical geoid model constructed from well geometrically distributed vertical control points, orthometric heights can be determined with uncertainties of about 1 cm±1 ppm. Differential geoid height uncertainties were tested all over the Eastern Region using the spirit levelling bench marks differential values, ellipsoidal heights differences and the EGM96 geoid model. The initial analysis of the first processing step revealed a difference between Al Ain Town Planning and Survey derived orthometric heights and the corresponding EGM96 heights of the magnitude of -0.35m. EGM96 values were then uniformly corrected by this regional default value. The mismatch between bias corrected EGM96 geoid values and the geometric geoid obtained from the differences between GPS ellipsoidal heights and orthometric heights was estimated within the expected tolerance.

1. Introduction:

Al Ain Town Planning and Survey Sector performed a WGS84 geometrical geoid processing for the determination of a local geoid covering the Eastern Region of Abu Dhabi Emirate to get use of the precise levelling coverage for orthometric height determination, and GPS observations to obtain WGS 84 coordinates of all control points in the region. The geoid is also computed from the earth geo-potential model (EGM96). The requirements of obtaining accurate GPS-derived orthometric heights has increased in recent years, intending to achieve the accuracy of orthometric height differences which will be viable alternative to traditional leveling techniques for many applications; now and in the years to come.
This paper may be considered as an attempt to drive orthometric heights with accuracies of about 2 to 5 cm from geometrical geoid determination. Investigation of the Eastern region height system and precise evaluation and analysis on the values of vertical control networks and benchmarks is outlined. Here considerations are to be taken for all bench marks, their original sources, information about the quality of such surveys. Then stability analysis is made based on precise levelling followed by GPS observation for all benchmarks. The WGS84 GPS/levelling geoid is computed as well as the global EGM96 global geoid. As many datums may be used for vertical control within Abu Dhabi Emirate, the geoid comparison between the GPS/Levelling geoid and the global geoid will give indications about the best network and hence the

vertical datum. Also assessment of Abu Dhabi vertical control datum and the relationship between the existing vertical control networks is made. Since all the height values within UAE are not related to one particular reference point, the paper pointed out that, care must be taken when using heights in the country, because heights datums may differ from one Emirate to another. Here the author calls for the unification of vertical datums within the country. In order to place individual surveying projects running a cross the country; into a uniform spatial context, a unified vertical and horizontal geodetic control network is vital. Such a network consists of a number of points spread across the area under consideration, these points are usually in the form of monumented benchmarks established and placed in the ground, along with a high-accuracy positional value for each point. Traditionally, in Al Ain there have been completely separate networks for horizontal and vertical control, but some and recent networks combine the two on common monuments. By referencing field measurements to such a network, the resulting data and information from multiple local survey activities can be accurately connected. The accuracy of each activity or project is no higher than the control network to which it is referenced. The most recent horizontal and vertical control networks of the Eastern Region are established by highly precise surveying methods followed by adjustment and statistical testing and analysis.

2. Geometric Geoid Determination

In recent years, modernization of vertical datums and associated height systems, have received considerable renewed attention world wide, particularly with regard to GPS-derived heights due to the use of GPS for precise positioning which in turn led to an increase in the use of refined geoid models for GPS height transformations and establishment of continuous operating reference stations. Many height systems have been defined for different vertical datums throughout the world, in UAE, most commonly known are Port Rashid and Ras Ghomis vertical datums. Each system has separate observation and computation procedures, data availability, accuracy requirements, compatibility with GPS, and the topographic settings in which the heights are used. Unfortunately even the relationship between these two datums is not well investigated.

Orthometric heights (h) are referenced to an equipotential reference surface known as the geoid, that approximating the mean sea level. The orthometric height of a point on the earth's surface is the distance from the geoid reference surface to appoint measured along the plumb line and vertical to geoid. While the ellipsoidal heights (H) are referenced to reference ellipsoid; and can be defined as the distance from the reference ellipsoid to the point, measured along the line which is normal to the ellipsoid. At the same point on the surface of the earth the difference between an ellipsoid height levels and orthometric height is defined as the geoid height, N, and can be approximated by:

$$N = H - h \qquad (1)$$

Errors that affect the accuracy of orthometric height, h, ellipsoidal height, H and geoid height, N, values are generally common to points near each other. Because the error sources are common, the uncertainty of height difference between near by points is significantly smaller than the uncertainty of the absolute height of each point. Orthometric height difference Δh_{AB} between two points A and B, can be obtained from ellipsoidal height differences ΔH_{AB} by substituting the geoid height differences ΔN_{AB};

$$\Delta h_{AB} = \Delta H_{AB} - \Delta N_{AB} \qquad (2)$$

The accuracy of Δh_{AB} is mainly dependent on the accuracy of ΔH_{AB} and ΔN_{AB}. Orthometric height differences, Δh_{AB}, can be obtained with uncertainly of about 1cm $\pm$ 1 ppm of the distance between the points i-e baselines less than l0 km were tested in the Eastern Region of Abu Dhabi Emirate and the orthometric heights differences were determined to $\pm$2cm level from GPS phase measurements. This accuracy will be increased in the near future by using Abu Dhabi GNSS Reference Station System (GRS) or by using spacing between local network stations not exceeding 10 km. This requirement will help in keeping the relative error in geoid height small, typically less than 5mm (Zilkoski, et.al 2005). Small values for the differential geoid height uncertainties have been demonstrated in the tests made in this investigation. These uncertainties can be further reduced by increasing the density of the vertical control network or by using gravity measurements and applying gravimetric geoid and adopting proper methods of gravity anomaly interpolation.

Determining uncertainly in geoid height differences, through a comparison of orthometric height and ellipsoidal height differences, depends on the error sources of levelling and ellipsoidal heights. Depending on the accuracy requirements, GPS surveys and high-resolution geoid models can be used instead of leveling methods.

In Al Ain, strategically all leveling benchmarks and previous horizontal control points were occupied by GPS and their 3-D geodetic coordinates were determined. The procedure applied in this investigation is based on the assumption that all differences between GPS observations on valid benchmarks need to agree within 2cm level. GPS-derived orthometric heights minus observed orthometric heights were computed.

Recently a total number of about 300 GPS/leveling benchmarks were measured in the Eastern region of Abu Dhabi Emirate, by Rolta Middle East associated with Geodelta. The measurements were performed using two sets of Leica dual frequency receivers 1200 and two DNK3 precise levels. All 300 GPS/Levelling benchmarks coordinates were computed on the known geodetic reference stations, using at least one hour GPS observations in order to define them in the same system at the specific epoch.

The combined use of global positioning system (GPS), leveling and geoid height information has been a key procedure in various geodetic applications. Although these three types of height information are considerably different in term of physical meaning, reference surface realization, observational methods, accuracy; etc. but they should fulfill the following geometrical relationship:

$$H - h - N = 0 \qquad (3)$$

Where H are ellipsoidal heights obtained from GPS observations, h are orthometric heights derived from leveling methods and N are the geoid heights computed from a geoid model. In practice equation (3) is never satisfied due to the random noise in values of H, h and N; datum inconsistencies and other possible systematic distortions; various geodynamic effects, and theoretical appropriations in the computation of ellipsoidal and geoid heights.

3. Geometrical Geoid Surface

The current height network in the Eastern Region is based mainly upon a series of leveling runs carried out by BKS Surveys between 1983 and 1987. A total of about, 214 levelling circuits were carried out over a total distance of 2396 km. The accuracy was specified as 10 mm per square root of levelling run in kilometres. The aim was to provide one point for every square kilometre in areas to be covered by 1:500 and 1:1000 mapping, and one point for every 5 km2 in areas to be covered by 1:2000 mapping. Of the 2000 ground markers, 814 were spirit levelled. The rest were heighted by reciprocal vertical angles. Many of these points have subsequently been destroyed during development work, and that only as few as 25% may still exist in some areas (Abdalla, 2006). The datum used for this levelling was the Port Rashid datum of Dubai, established by levelling carried forward from Dubai to points BTP219 (Al Faqa) and BTP226 (Schweib). These two points mark the northern extent of the levelling work; the southern extent is at Al Qua. In addition to the levelling and trigonometric heighting established by BKS, two other sources of height information are available in Al Ain Town planning and survey sector database. The first of these are the points identified in the database by the initial prefix "K", which relate to a boundary survey carried out along the border with Oman by KLM. The second set of points is that identified by the initial prefix "G", which were geodetic points established by the Survey Section of Al Ain Town Planning in the 1990s. Concrete monuments were installed in all non-rocky and non-sandy locations. In rocky locations where excavation proved difficult a shortened central tube was used. In sandy locations liable to erosion, the three-metre pipe marker was installed with witness posts.

In the future, the transformation between horizontal datums may become unnecessary, as most countries move to use WGS84 as a geocentric datum for surveying and mapping, as in case of UAE. But the ellipsoidal heights are not usually used directly for practical surveying, engineering or geophysical applications as they have no physical meaning (Featherstone, et.al 1998). So it will be necessary to transform GPS-derived ellipsoidal heights to orthometric heights, using the relation of the geoid with respect to the WGS 84 ellipsoid.

Usually the geoid is used as the reference surface to orthometric heights, and the vertical datum is assumed to be established using tide gauge measurements of mean sea level in conjunction with geodetic leveling. In the Abu Dhabi Emirate, orthometric heights are assumed to be referred to Ras Ghomis vertical datum, but the coincidence of the vertical datum with the geoid surface needs further investigation.

There are two main approaches to transform ellipsoidal heights to orthometric heights, namely gravimetric method and geometrical method. In this paper, only the geometric method of geoid determination is considered. Ellipsoidal and orthometric heights are given at about 300 benchmarks to provide the geoid undulations at these common points. A simple geoid surface is created and the geoid contour is shown in fig (1). A comparison is made using a linear interpolation method to derive the geometrical WGS84 geoid undulation. The basic interpolation method used to derive orthometric heights from GPS observations is illustrated in equation (4). For example, If two points A and B are considered, with their known 3-D coordinates (E,N,H) and orthometric heights h, the geoid undulations for any point X observed by GPS between points A and B; can be given as (Featherstone et.al 1998):

$$h_x = h_A + \Delta H_{AX} - \frac{L_{AX}}{L_{AB}} \Delta N_{AB} \quad (4)$$

Where L_{AX}, L_{AB} is the distances between point A and points X and B respectively. The geometric determination of orthometric height using this approach is very useful especially for short baselines. For areas where the bench marks are well geometrically distributed, a plane surface is created using the following formula (Featherstone, et.al 1998):

$$H - h = N = \alpha_0 + \alpha_1 E + \alpha_2 N \quad (5)$$

where α_0 is the coefficient representing the bias, α_1 and α_2 are the coefficients represent tilts of the geoid plane will respect to WGS84 ellipsoid, and E, N are the easting and northing coordinates. To determine the three coefficients α_0, α_1 and α_2, GPS observations at a minimum of three bench marks (A, B and C) surrounding the area under consideration should be made, then applying the following equation:

$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 & E_A & N_A \\ 1 & E_B & N_B \\ 1 & E_C & N_C \end{bmatrix}^{-1} \begin{bmatrix} (H-h)_A \\ (H-h)_B \\ (H-h)_C \end{bmatrix} \qquad (6)$$

The coefficients α_0, α_1 and α_2 can be determined using equation (6), and then used in equation (7) to determine the orthometric height of any point observed by GPS in the surrounding area.

$$h_X = h_A + \Delta H_{AB} - N_A + \alpha_0 + \alpha_1 E_X + \alpha_2 N_X \qquad (7)$$

There are some important precautions that must be taken when using geometrical interpolation to determine orthometric heights from GPS, these include:

(1) To maximize the accuracy of ellipsoidal and orthometric heights.
(2) Geometrical geoid can be superior to gravimetric geoid for survey areas that use smaller than the resolution of the gravimetric geoid.
(3) Interpolation method should be applied in the area bound by the bench marks used to define the surface.
(4) Independent checks should always be used to ensure that the geoid is accurately modeled.

The ultimate aim for this investigation is to determine a geometrical geoid surface across the region, in such a way that GPS observations can be corrected so that they agree with the orthometric height datum. An initial assessment of problems associated with the geoid in Al Ain region were made by Hansa Luftbild (Abdalla 2006); followed by this attempt by using the observed WGS-84 coordinates of almost all control points across the region. From the differences between ellipsoidal and orthometric heights, the values for the geoid heights were determined. These values were then compared with the value given by the EGM-96 global Earth model, and the results are shown in Table (1). After correcting for an overall bias (which would be expected since the heights use different datums), the residual variations are carefully studied. However, considering the known accuracy of EGM-96, the type of terrain, and the area covered, these variations are within the expected values (see table. 1).

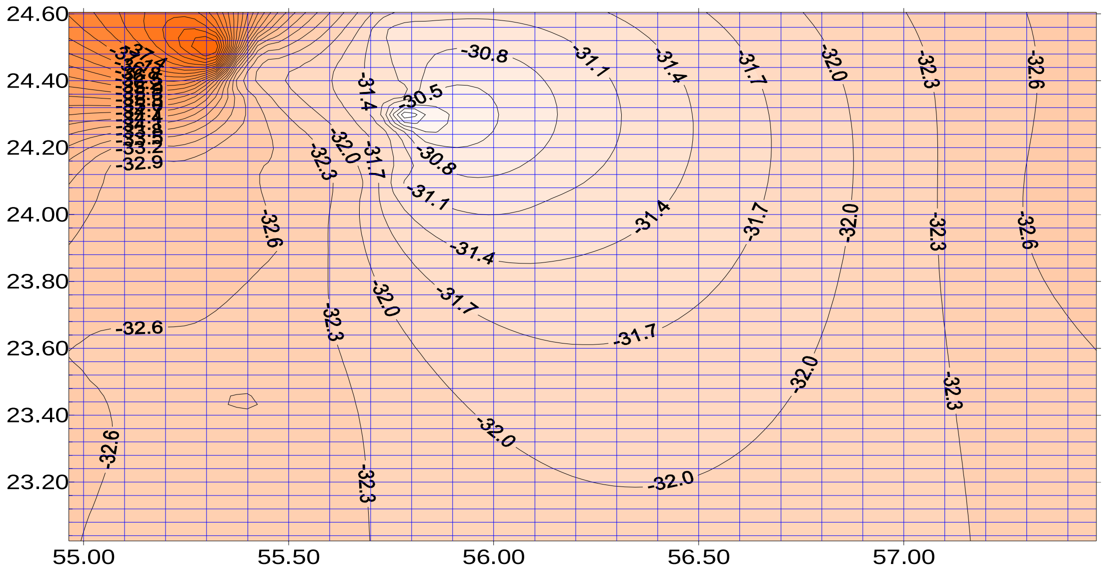

Fig.(1): WGS84 Geoid Contour of the Eastern Region

4. The Earth Geo-potential Model EGM96

In this study the EGM96 model of the Earth which is covering the Earth gravity field up to degree and order 360 is used. This corresponds to model the geoid within an accuracy of about 40 cm (global average). For a further refinement of the EGM96 in the Eastern Region, surface point information has to be introduced. A point separation of about 10 km is needed to cover the short wave length part of the harmonic development of the Earth gravity field. The following illustration shows the part of the EGM96 model that covers the Eastern region of Abu Dhabi Emirate. In the studied area, a steep increase of the geoidal height of about 3m (from -33.24m to -30.6m) occurs over distance of about 100km. The initial analysis of the first processing step revealed a difference between orthometric heights and the corresponding EGM96 heights of the magnitude of 0.8m (table.2). EGM96 values were then uniformly corrected by this regional default value. However, the mismatch between bias corrected EGM96 values and actual GPS height minus orthometric heights was in the range of 50 cm.

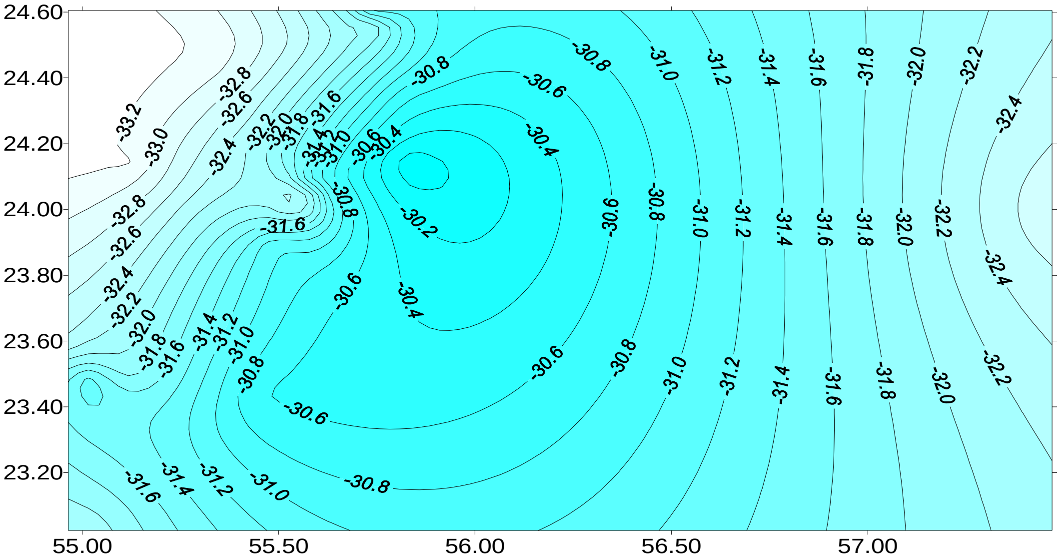

Fig. (2): EG96 Geoid Contour of the Eastern Region

The discrepancy between the EGM96 model (bias corrected) and the observations was agreed within ±40 or ±50 cm as expected, considering the overall quality of the EGM96 model. At the present state of data evaluation it is not possible to provide reliable geoid information for the Eastern Region of Abu Dhabi Emirate, which can be applied in civil engineering. This showed that there is an urgent need for having precise geoid information that will exploit the full potential of the GPS surveying technique by extending it use to all three-dimensions, especially after the full implementation of Abu Dhabi GRS system. This approach is considered as a fundamental component of determining a local geoid using an accurate homogeneous control network extending throughout the region. The exploitation of this valuable resource to define the relationship between the heights from GPS surveys and the orthometric heights is one of the main objectives of this study, hence allowing the various user groups of spatial data to maximize their investment in GPS technology.

In planning new geodetic measurements for the establishment of a comprehensive and accurate geoid, special attention will have to be given to the resolution of grid network for observed gravimetric or levelling data, which will be the basis for a local geoid refinement. Accurate and reliable GPS/levelling stations along the boarder of the region and inside the area are necessary to overcome uncertainties in the global model. The extent of the surveying activities within the region will be dependent on the uncertainties required within the local geoid model.

Station	Geoid Undulation [m]		Residuals [m]
	Observed	EGM96	Observed Geoid minus EGM96.
A134	-30.806	-30.73	-0.076
B180	-32.626	-32.64	0.014
B191	-32.904	-32.83	-0.074
B263	-32.987	-32.99	0.003
B266	-32.925	-32.91	-0.015
B273	-32.853	-32.78	-0.072
B290	-32.715	-32.43	-0.285
B295	-32.646	-32.28	-0.366
B314	-32.441	-31.75	-0.691
B340	-32.652	-32.27	-0.382
B94	-30.99	-31.73	0.74
BM622	-32.985	-32.96	-0.025
F330	-32.915	-33.14	0.225
G6130	-32.793	-32.83	0.037
G6134	-32.774	-32.22	-0.554
G6148	-32.628	-32.43	-0.198
G6170	-32.582	-31.7	-0.882
G6171	-32.59	-31.95	-0.64
G6172	-32.58	-32.14	-0.44
G6173	-32.591	-32.37	-0.221
G6174	-32.646	-32.49	-0.156
G6175	-32.687	-32.53	-0.157
G6176	-32.742	-32.69	-0.052
G6177	-32.754	-32.76	0.006
G6387	-32.865	-32.64	-0.225
G6433	-31.271	-30.62	-0.651
G6470	-30.877	-30.69	-0.187
G6474	-31.082	-30.84	-0.242
G6476	-31.535	-30.9	-0.635
G6506	-30.825	-30.97	0.145
G6511	-32.792	-32.49	-0.302
G6522	-31.093	-31.87	0.777
G6527	-31.604	-32.02	0.416
G6530	-32.344	-31.64	-0.704
G6531	-32.776	-32.59	-0.186
G6544	-32.662	-32.35	-0.312
G6545	-32.749	-32.51	-0.239
G6546	-32.856	-32.78	-0.076
G6547	-32.902	-33.1	0.198
G6548	-32.957	-33.25	0.293
G6561	-32.9	-33.24	0.42
G6564	-32.149	-32.32	0.171
G6570	-31.896	-32.18	0.284
G6571	-31.37	-31.94	0.57
G6574	-32.9	-33.06	0.16
G6591	-32.718	-32.19	-0.528
H3201	-32.236	-31.36	0.876
T3723	-32.424	-32.49	0.066

Table (1): WGS 84 Geometrical and EGM 96 geoid at common points

Common Stations	Height Differences (m)	Common Stations	Height Differences (m)
A134	-0.9834	G6470	-0.7195
B1221	-0.936	G6474	-0.4716
B1252	-0.925	G6476	-0.792
B1260	-0.93	G6486	-0.903
B1650	-1.137	G6506	-0.5676
B1692	-1.116	G6531	-1.1183
B180	-0.725	G6514	-0.8401
B191	-0.758	G6522	-0.8464
B263	-0.757	G6527	-0.8286
B266	-0.767	G6530	-1.0399
B273	-0.882	G6531	-1.1183
B290	-0.885	G6544	-1.1512
B295	-0.786	G6545	-1.3066
B314	-1.077	G6546	-0.833
B340	-1.525	G6547	-0.7651
B743	-0.924	G6548	-0.7627
B94	-0.785	G6561	-0.6028
BM621	-0.782	G6564	-0.7932
G6130	-2.649	G6570	-0.9659
G6134	-2.457	G6571	-0.7053
G6166	-1.1373	G6574	-0.8758
G6167	-1.4704	G6591	-1.1436
G6387	-0.938	H3201	-1.2004
G6431	-0.8451	T3723	-0.6999
G6433	-0.847	G6470	-0.7195
G6514	-0.8401	G6474	-0.4716
G6522	-0.8464	G6476	-0.792
G6527	-0.8286	G6486	-0.903
G6530	-1.0399	G6506	-0.5676

Table (2): Height differences based on Ras Ghomis and Port Rashid UAE Vertical Datums

Greater height accuracies will be required within the developed and developing areas and therefore smaller uncertainties will be required within the local geoid within these areas. Outside these areas a less rigorous determination of the model will be required, which can be updated inline with the ever expansive survey requirements

The analysis of the two processing steps adopted revealed expected differences between EGM96 values and actual GPS height minus orthometric height values. As a consequence it can be concluded that given the present set of data is not possible to provide reliable geometrical geoid information for the whole of the Eastern Region of Abu Dhabi Emirate that can be applied in civil engineering.

This study can be considered as an attempt to drive orthometric heights with accuracies of about 2 to 5cm from geometrical geoid determination. Investigation of the Eastern region height system and precise evaluation and analysis on the values of vertical control networks and benchmarks revealed that a precise gravimetric geoid should be established for the aid of using GPS for orthometric height determination. The stability analysis of the precise levelling followed by GPS observation for all monumented benchmarks illustrated that, the geometrical geoid can be used provided that the geoid surface created should be based on well distributed short resolution control points.

5. Conclusions

The paper showed that the orthometric heights in the Eastern Region of Abu Dhabi Emirate will be based on Ras Ghomis vertical control datum. The mean accuracy of the vertical control network in the region is within 3cm. By applying the geometrical geoid model, orthometric height differences can be obtained with uncertainly of about 1cm $\pm$ 1 ppm of the distance between the points.

The discrepancy between the EGM96 model (bias corrected) and the observations was agreed within ±50cm, considering the overall quality of the EGM96 model. The initial analysis of the first processing step revealed a difference between orthometric heights and the corresponding EGM96 heights of the magnitude of 0.3m.

The study recommended that Al Ain Town Planning and survey sector has to initiate a new project for short wave length gravimetric and geometrical geoid determinations to cover at least the entire build up areas of the eastern region of Abu Dhabi Emirate.

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