Use Of Similarity Transformations To Improve GPS Heighting

M.N. J.P. Vella
Centre for Geodetic and Geodynamic Studies (CGGS)
Faculty of Geoinformation Science and Engineering
University of Technology Malaysia
Johor, Malaysia

ABSTRACT

Corrector surfaces have been used to model the differences between the geoid and the mean sea level derived orthometric heights, which are not coincident due to varying factors. In this case corrector surfaces are derived for 25 points co-located with GPS ellipsoidal heights, orthometric heights and geoid derived gravimetric heights, which lie throughout Johor, the southern State of Peninsular Malaysia. In order to model and minimise the differences a similarity transformation is utilised for fitting the gravimetric co-geoid undulations with GPS/Levelling data. Before the transformation the comparison between GPS and levelling is a standard deviation of ± 0.151 m and a mean of 0.699 m. When tested the internal relative accuracy suggests a standard deviation of ± 0.072 m and a mean of 0.003 m, which indicates the bias between the two datum's has been eliminated. With the valid transformation it is now possible for the GIS field user to apply accurate corrected geoid heights to GPS derived ellipsoidal heights in a rapid feature driven field mode.

1 INTRODUCTION

Orthometric heights traditionally are determined through optical methods involving the transfer of height difference from a datum point to the unknown point, where the orthometric height is required. This can sometimes be a very arduous task and now with the advent and proliferation of the use of the Global Positioning System (GPS), this task realistically seems more possible now than ever before. With the use of GPS and a regional gravimetric geoid model it is possible to transfer heights simply, as the following relationship demonstrates in an absolute sense:

$$H_A = h_A - N_A \tag{1}$$

or

$$h_A - H_A - N_A = 0$$

This relationship shows how the orthometric height H_A is related to the geometrical ellipsoidal height obtained from GPS measurements h_A and the physical geoid/ellipsoid separation N_A. The relationship however, is not always appropriate due to the physical way in which GPS surveys are conducted; in general it is more suited to use the following relative case of Eq. (1).

$$\Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \tag{2}$$

Eq. (2) shows the relationship with respect to relative differences for the orthometric heights H_A and H_B, the ellipsoid heights h_A and h_B and the physical geoid/ellipsoid separations N_A and N_B. This relationship allows differential GPS measurements to be used, which are known to be more precise and in conjunction to this geoid models are now becoming more precise and are constantly pushing the 1cm level of accuracy. Having such accurate information in the form of the geoid and ellipsoid heights enables Geomatics Engineers and other research Scientist to apply this information in their respective fields thus realistically providing the opportunity for GPS to be used in GPS levelling and other applications.

Studies carried out on the geoid and GPS/Levelling, in different countries show that GPS and the geoid are now more than ever important tools, such studies are (Kotsakis and Sideris.,, 1999), (Mainville et al., 1997), (Zhong., 1997) and (Martensson, 2002).

In Malaysia computing the geoid has been of prime interest in the past and geoid models have been computed for either the whole of Peninsular Malaysia or a part thereof, see (Kadir et al., 1999) and (Vella et al., in press). Peninsular Malaysia is a country traversed north and south by very rugged mountain ranges that have largely prevented access to the hinterland for conventional terrestrial gravity surveys. All previous attempts at computing the geoid in Peninsular Malaysia have suffered from lack of data and non-homogeneity of the data distribution. However the Department of Surveying and Mapping Malaysia (DSMM) have embarked upon a very ambitious project to collect new gravity data and update the existing database through the use and implementation of airborne gravity surveys. This provides the impetus for the current study in that the new data will provide new geoid models, which in turn will benefit from studies showing which is the most appropriate technique to apply for corrector surfaces (CS) in Peninsular Malaysia. Corrector surfaces need to be applied as the relationship in Eq. (1) is rarely satisfied, reasons for this are as described in (Kotsakis and Sideris., 1999). These might be, (1) random noise in the ellipsoidal heights, orthometric heights or geoid/ellipsoid separations, (2) datum inconsistencies and other systematic distortions, (3) various geodynamic effects and (4) theoretical approximations in the computation of either the orthometric height (H) or the geoid/ellipsoid separation (N) are not good enough. Peninsular Malaysia lies well away from any major subduction zones, the possible reasons for (3) could well be land subsidence at tide gauges used for the vertical datum and or mean sea level rise. This however is not investigated here and is beyond the scope of this work.

2 EVALUATION OF GEOID MODELS AND GPS DATA

DSMM kindly provided data for the southern region of Peninsular Malaysia, containing geodetic latitude, longitude, orthometric height and ellipsoidal height. The geoid heights were provided from independent sources namely (Kadir et al., 1999), (Vella et al 2003) and EGM96 geopotential model coefficients by (Lemoine et al, 1997).

2.1 SOUTH EAST ASIAN CO-GEOID

This co-geoid is a high resolution and high precision model computed for the whole of South East Asia (Kadi et al, 1999). The EGM96 geopotential model truncated to degree and order 70 was combined with surface gravity data and a modified Stokes's kernel was used in the computations. Comparisons of the gravimetric geoid with the GPS/Levelling derived geoid/ellipsoid separations at 140 points throughout Peninsular Malaysia show that the absolute agreement with respect to the GPS/Levelling datum is generally better than 40 cm RMS, which demonstrates an improvement over EGM96 and OSU91A geoid models for the same points. For ease of reference this co-geoid model will be referred to as CG1.

2.2 PENINSULAR MALAYSIA CO-GEOID

Free-air gravity anomalies are usually found to be locally correlated with the elevation of observing points. The correlation between the observed height of the free-air anomaly and its corresponding value is derived using least squares. It is shown that a minimum height of 400 m yields the best correlation (R=0.886) between the free-air anomaly and the height and therefore this height of 400 m is used as a minimum height to interpolate data from the GTOPO30 global Digital Elevation Model (DEM). From the DEM, all heights with a height equal to or greater than 400m, are used to derive anomaly-height correlated values. When these values are combined with the original gravity data set and gridded, the gridding is better controlled, as results over Peninsular Malaysia show. Using the 1D FFT and stokes integral with no modification the co-geoid is computed, (Vella et al., in press). The resulting free-air co-geoid model is compared to 143 GPS points. The model attained an RMS of 46.1 cm when compared at the GPS points showing an improvement over EGM96. For ease of reference this co-geoid model will be referred to as CG2.

2.3 EGM96 COMPARISONS

The NASA Goddard Space Flight Center, The National Imagery and Mapping Agency (NIMA) and the Ohio State University (OSU) have collaborated to produce EGM96, an improved degree 360 spherical harmonic model representing the earth's gravitational potential (Lemoine et al, 1997). The comparisons between the EGM96 model and the difference between the orthometric height and ellipsoidal height show that the RMS is 51cm at 143 GPS points.

3 OVERVIEW OF MODELLING CONSIDERATIONS FOR THE CS

Presented herein are the basic models describing mathematically the different schemes used in the adjustment and transformation results. There are other models and methods which account for local deformations in the geoid and other error sources, that can be used for corrector surfaces as demonstrated by (Zhong, 1997), (Featherstone, 1998) and (Zhiheng and Duquenne., 1996).

3.1 SIMILARITY TRANSFORMATIONS

The basic model used is of a modified form of Eq. (1) as follows:

$$h_i - H_i - N_i = c_i^T x + v_i \quad (3)$$

where h_i, H_i, and N_i are as previously described, $\boldsymbol{x}$ is an $n \times 1$ vector of unknown parameters, a_i is an $n \times 1$ vector of known coefficients, and v_i is the residual random noise term, (see e.g. Kotsakis and Sideris 1999). As stated in the introduction test are conducted on three similarity transformation schemes and four polynomial schemes, all schemes are solved using parametric least squares techniques according to Eq. (3) but each with differing observation Eq.s, for obvious reasons.

It has been widely held that the four parameter model is best suited to this type of modelling, this however is not necessarily the case as the results will show, however it is important to state clearly all the models and there mathematical representation. The four parameter (Eq. 4) model from now on called CS4, is an approximate similarity transformation model describing the geoid undulation transformation; this scheme is adequately discussed in Heiskanen and Moritz (1967, Sect. 5-9):

$$\mathbf{c}_i^{\mathrm{T}}\mathbf{x} = \cos\varphi_i \cos\lambda_i \, x_1 + \cos\varphi_i \sin\lambda_i \, x_2 + \sin\varphi_i \, x_3 + x_4 \quad (4)$$

The five parameter scheme (CS5), a rigorous similarity transformation model is also discussed in Heiskanen and Moritz and is described as follows:

$$\mathbf{c}_i^{\mathrm{T}}\mathbf{x} = \cos\varphi_i \cos\lambda_i \, x_1 + \cos\varphi_i \sin\lambda_i \, x_2 + \sin\varphi_i \, x_3 + \frac{\sin\varphi_i \cos\varphi_i \sin\lambda_i}{W_i} x_4 + \frac{\sin\varphi_i \cos\varphi_i \cos\lambda_i}{W_i} x_5 \quad (5)$$

The eight parameter scheme (CS8), which is a rigorous non-rigid similarity transformation model, is described as follows:

$$\mathbf{c}_i^{\mathrm{T}}\mathbf{x} = \cos\varphi_i \cos\lambda_i \; x_1 + \cos\varphi_i \sin\lambda_i \; x_2 + \sin\varphi_i \; x_3 + \frac{\sin\varphi_i \cos\varphi_i \sin\lambda_i}{W_i} x_4 + \frac{\sin\varphi_i \cos\varphi_i \cos\lambda_i}{W_i} x_5 + (a W_i + h_i)\, x_6 + \frac{1-f^2 \sin^2\varphi_i}{W_i} x_7 + \frac{\sin^2\varphi_i}{W_i} x_8 \tag{6}$$

where the quantity W_i is given by the relationship

$$W_i = \sqrt{1-e^2 \sin^2\varphi_i} \tag{7}$$

and the quantities f , a and e in the above formulas correspond to the flattening, the semi-major axis and the first eccentricity, respectively, of the reference ellipsoid (either the ellipsoid used for the GPS heights, or the ellipsoid used for the gravimetric geoid model).

3.2 POLYNOMIAL MODELS

The first degree polynomial (CS3) is a three parameter scheme described as follows:

$$\mathbf{c}_i^{\mathrm{T}}\mathbf{x} = (\lambda_i - \lambda_o)\, x_1 + (\varphi_i - \varphi_o)\, x_2 + x_3 \tag{8}$$

The second degree polynomial (CS6) is a six parameter scheme described as follows:

$$\mathbf{c}_i^{\mathrm{T}}\mathbf{x} = (\lambda_i - \lambda_o)\, x_1 + (\varphi_i - \varphi_o)\, x_2 + x_3 + (\lambda_i - \lambda_o)^2 x_4 + (\varphi_i - \varphi_o)^2 x_5 + (\lambda_i - \lambda_o)(\varphi_i - \varphi_o)\, x_6 \tag{9}$$

The third degree polynomial (CS10) is a ten parameter scheme described as follows:

$$\mathbf{c}_i^{\mathrm{T}}\mathbf{x} = (\lambda_i - \lambda_o)\, x_1 + (\varphi_i - \varphi_o)\, x_2 + x_3 + (\lambda_i - \lambda_o)^2 x_4 + (\varphi_i - \varphi_o)^2 x_5 + (\lambda_i - \lambda_o)(\varphi_i - \varphi_o)\, x_6 + (\lambda_i - \lambda_o)^3 x_7 + (\varphi_i - \varphi_o)^3 x_8 + (\lambda_i - \lambda_o)^2 (\varphi_i - \varphi_o)\, x_9 + (\lambda_i - \lambda_o)(\varphi_i - \varphi_o)^2 x_{10} \tag{10}$$

Fourth degree polynomial (CS15) is a fifteen parameter scheme described as follows:

$$
\begin{aligned}
\mathbf{c}_i^{\mathrm{T}}\mathbf{x} = {} & (\lambda_i - \lambda_o)\,x_1 + (\varphi_i - \varphi_o)\,x_2 + x_3 \\
& + (\lambda_i - \lambda_o)^2\,x_4 + (\varphi_i - \varphi_o)^2\,x_5 + (\lambda_i - \lambda_o)(\varphi_i - \varphi_o)\,x_6 \\
& + (\lambda_i - \lambda_o)^3\,x_7 + (\varphi_i - \varphi_o)^3\,x_8 + (\lambda_i - \lambda_o)^2(\varphi_i - \varphi_o)\,x_9 \\
& + (\lambda_i - \lambda_o)(\varphi_i - \varphi_o)^2\,x_{10} + (\lambda_i - \lambda_o)^4\,x_{11} + (\varphi_i - \varphi_o)^4\,x_{12} \\
& + (\lambda_i - \lambda_o)^3(\varphi_i - \varphi_o)\,x_{13} + (\lambda_i - \lambda_o)(\varphi_i - \varphi_o)^3\,x_{14} \\
& + (\lambda_i - \lambda_o)^2(\varphi_i - \varphi_o)^2\,x_{15}
\end{aligned} \tag{11}
$$

In the above formulas φ_o and λ_o denote the average latitude and longitude, respectively, of the test area

The schemes describing CS4,CS5,CS8,CS3,CS6,CS10 and CS15 are applied to all points in the network, in this case 25 points and are operated on by least squares in order to minimise the residuals, thus giving the best estimate of the different coefficients involved.

The adjustment carried out of the coefficients is in no way optimal as pointed out by (Kotsakis and Sideris, 1999), there is no weighting of the residuals done, thus leaving some questionable doubt as to which term is contributing to the residual, whether it is the ellipsoidal height, the orthometric height or the geoid/ellipsoid separation, it may even be a combination of some or all of these terms. So although some of the schemes may be rigorous, the solution is in no terms a rigorous solution, in this case.

4 TESTS CARRIED OUT ON CONTROL DATA

As previously mentioned the control GPS points are only located in the state of Johor, whereas the comparisons mentioned in Sections 2.1, 2.2 and 2.3 encompass the whole of the Peninsular Malaysia. So as to prevent any confusion the final comparisons will only include the 25 points for the state of Johor.

Before any schemes were tested all the GPS points were screened to make sure they all fall within three standard deviations of the mean of differences between N_{EGM96} derived using EGM96 and $N_{\text{h-H}}$ derived using *h* and *H*. This is a quick and dirty data snooping technique and since there were no outliers there was no statistical reasoning to exclude any points although when compared the residual can be large. Table 1 shows the statistics of the comparisons from the residuals concerned with the above discussion.

No. Stns	25
Max	63.0
Min	-35.3
Mean	71.2
Stnd Dev ±	23.2

Table 1: Statistics of the residuals for the comparison of the outlier test conducted between N_{EGM96} and (h-H) ie: [(h – H) – N_{EGM96}], (All units are in cm).

The largest residual is 0.653m at TG05, located in the far north of the state, there are also other points indicating large residuals and if these points along with the respective residuals are plotted as in Figure 1, it is possible to identify visually which points could be excluded in order to improve the results of the corrector surfaces. As better data in the adjustment will give better-adjusted coefficients, however there is the risk of excluding perfectly good data, for this study we have included all data, except for some data excluded for the purpose of external assessment of the adjustment of the coefficients.

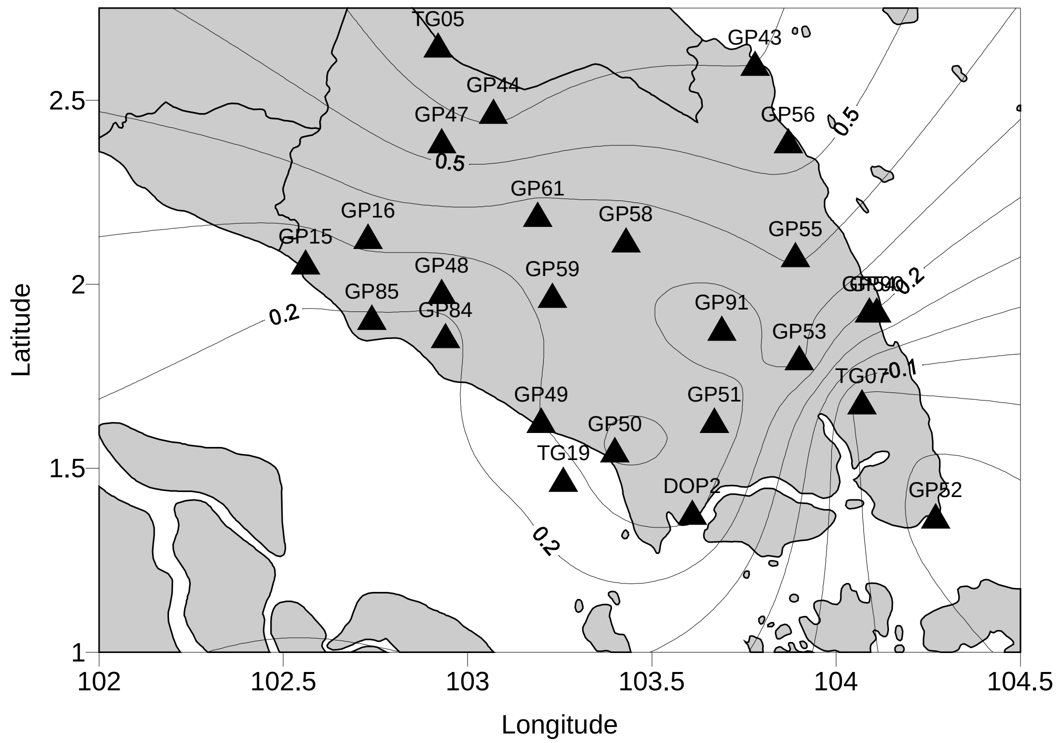

Figure 1: Location of GPS points (25) showing residual contour plot $[(h - H) - N_{EGM96}]$, contour interval 10cm

It is evident in Figure 1, that the further north the points progress the larger the residuals $[(h - H) - N_{EGM96}]$ become, this perhaps is indicative of some systematic bias in the Peninsular Malaysia vertical datum, or it could just show there are problems in the adjustment of the vertical datum, this however is not investigated here.

5 RESULTS FROM COMPARISONS OF CORRECTOR SURFACES

Two types of comparisons are made, (1) all 25 GPS data points are used in the adjustment and the resulting residuals, Eq (3) are investigated, (2) 3 GPS points are excluded from the adjustment, namely GP48, GP58 and GP51 and are compared with the adjusted coefficients, this provides an external comparison of how well the coefficients are able to account for any deformations and dissimilarities there are between the GPS, vertical datum and co-geoid models used. Adjustments will be made using both available co-geoid models (CG1 & CG2) for Peninsular Malaysia.

5.1 ADJUSTMENT RESULTS OF CORRECTION SURFACES

Comparisons are carried out using the following for original misclosures and adjusted residuals respectively in Eq (12) and Eq (13).

$$h_i - H_i - N_i = l_i \qquad (12)$$

and

$$v_i = l_i - c_i^T x \qquad (13)$$

In the following tables No CS refers to no correction surface is applied and the original misclosures statistics are listed using (Eq 12), where as for the correction surfaces Eq (13) is being employed and the statistics all represent those of the residuals from the adjustment. Table 2 shows the statistics for CG2 and the results of the adjustments.

	No CS	CS4	CS5	CS8	CS3	CS6	CS10	CS15
# pts	25	25	25	25	25	25	25	25
Max (cm)	92.2	27.2	27.3	15.6	32.2	19.2	16.4	19.5
Min (cm)	-6.3	-34.3	-32.3	-18.3	-38.3	-26.1	-26.7	-24.4
Mean(cm)	57.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Std± (cm)	22.7	13.5	15.2	7.3	18.3	11.9	9.2	8.3
RMS (cm)	61.9	13.2	14.9	7.2	17.9	11.6	9.0	8.1

Table 2: Statistical summary using CG2 to derive the coefficients for the similarity transformation and polynomial fitting at 25 GPS points.

	No CS	CS4	CS5	CS8	CS3	CS6	CS10	CS15
# pts	25	25	25	25	25	25	25	25
Max (cm)	29.0	20.6	18.4	12.7	25.5	12.3	14.8	18.2
Min (cm)	-71.5	-37.7	-32.6	-16.4	-41.5	-28.1	-27.4	-21.8
Mean(cm)	-1.8	0	0.0	0.0	0.0	0.0	0.0	0.0
Std± (cm)	25.8	13.5	11.4	7.0	15.4	9.0	8.3	7.1
RMS (cm)	25.4	13.2	11.2	6.8	15.1	8.8	8.2	7.0

Table 3: Statistical summary using CG1 to derive the coefficients for the similarity transformation and polynomial fitting at 25 GPS points.

Evidently, from Table 2, it is clear that the corrector surface CS8 has managed to remove the bias term, as do all corrector surfaces, however CS8 has achieved the smallest standard deviation of the adjusted residuals. Using no corrector surface (No CS) shows how there is still a great amount of variability between the different components (ie: *h, H and N*) for Peninsular Malaysia. Table 3 shows the same results only this time for CG1. The results from CG1 clearly improve, although only slightly but it seems that CG1 is a better fit to the local GPS data than that of CG2, as is indicated by its reduced mean (-1.8cm).

5.2 ADJUSTMENT RESULTS FOR NATIONAL COMPARTMENTS

Figure 2, shows the national distribution of 140 GPS points, it is clearly seen from Figure 2 that the hinterland is largely uncovered by any kind of geodetic information, be it gravity, GPS or levelling. When test are carried out for the whole 140 GPS points distributed nationally through out Peninsular Malaysia, it is evident as is shown in Table 4, that the fit of the CG2 to the national data set is not too good. The mean of 0 indicates that all the schemes are capable of removing any bias however the large standard deviations indicate there are still discrepancies between the three components, *h, H* and *N*. These discrepancies may be due to systematic biases, or simple errors in the data and the results from Tables 2 and 3 would tend to suggest they are geographically correlated, in these Tables only 25 data points for Johor were used as opposed to the data set containing 140 data points for the whole of the Peninsular Malaysia.

	No CS	CS4	CS5	CS8	CS3	CS6	CS10	CS15
# pts	140	140	140	140	140	140	140	140
Max (cm)	86.6	83.0	73.9	69.1	83.0	78.4	75.1	64.4
Min (cm)	-90.2	-79.3	-75.1	-56.1	-86.1	-79.0	-58.9	-55.3
Mean(cm)	6.8	0	0.0	0.0	0.0	0.0	0.0	0.0
Std± (cm)	38.3	26.72	26.4	24.1	26.8	26.4	23.4	22.8
RMS (cm)	38.8	26.6	26.3	24.0	26.7	26.3	23.3	22.8

Table 4: Statistical summary using CG2 to derive the coefficients for the similarity transformation and polynomial fitting at 140 GPS points.

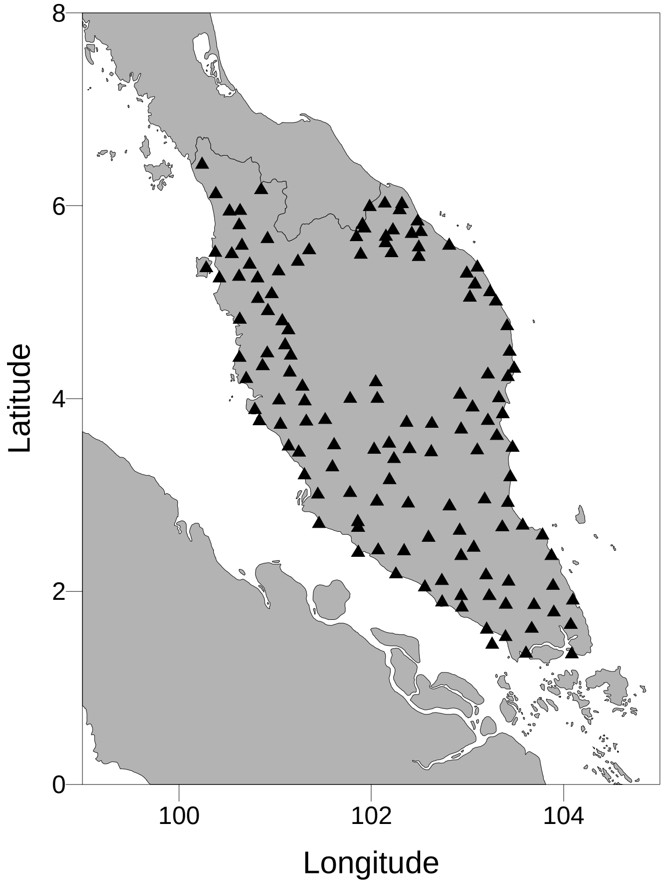

Figure 2: Distribution of 140 GPS points for Peninsular Malaysia

In order to determine if this is so, tests are conducted were the whole data set is broken down into individual compartments based on States, as is shown in Figure 3.

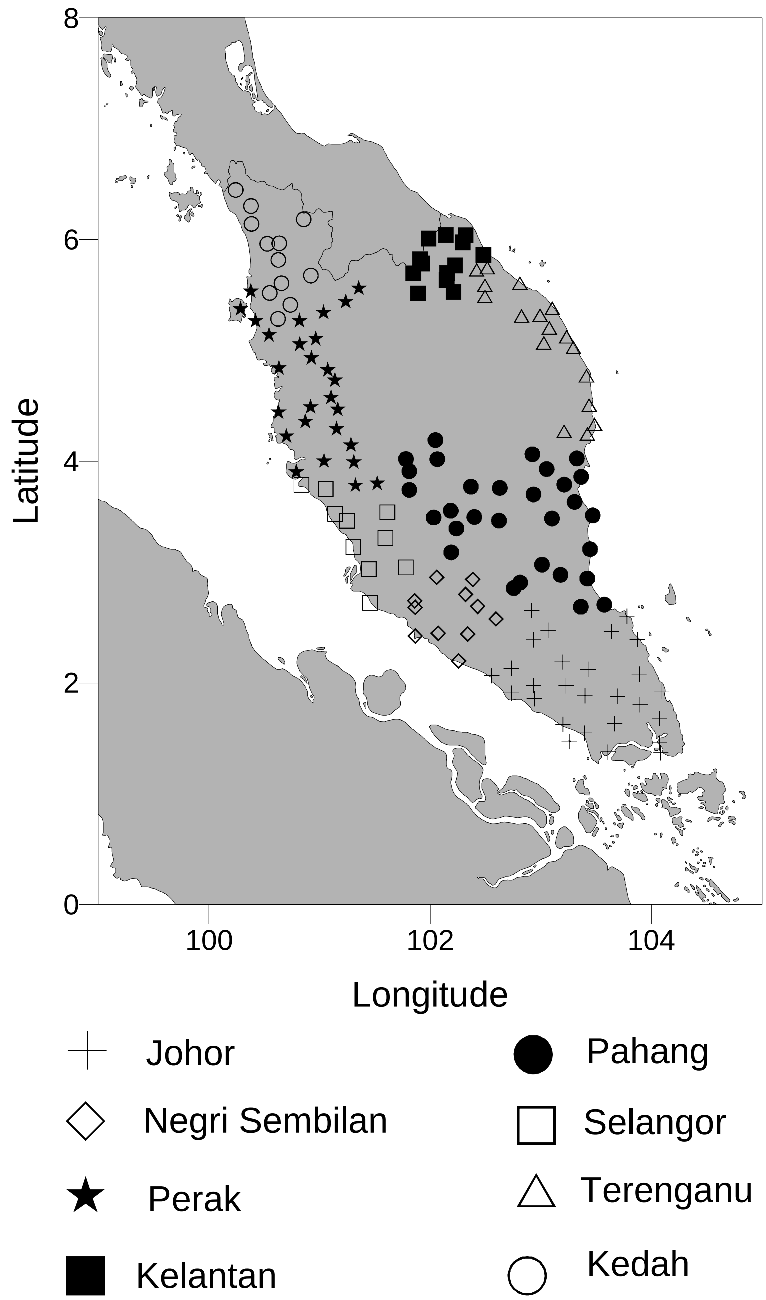

Figure 3: Distribution of 140 GPS points for Peninsular Malaysia, based on their respective State

This is similar to what was conducted in France, (eg. Zhiheng and Duquenne, 1996) where the geoid was divided up into smaller pieces and then adjusted to the GPS levelling with constraint conditions applied, in this case there will be no constraint conditions applied. This is due largely to the fact that the necessary information is not available, therefore it is considered a semi-rigorous attempt to model the spatial differences there may be between the three components of the observation equations. Tables 5 through to Table 10 , show the results for all States included, and only with the eight parameter transformation, CS8 as this has already proven to be a well developed scheme, see Tables 2 and 3.

	No CS	CS8
# pts	10	10
Max (cm)	37.7	10.1
Min (cm)	-55.0	-9.5
Mean(cm)	-18.3	0.0
Std± (cm)	28.0	5.4
RMS (cm)	32.3	5.1

Table 5: Statistical summary using CG2 to derive the coefficients for the similarity transformation CS8 at 10 GPS points, for Selangor.

	No CS	CS8
# pts	26	26
Max (cm)	7.2	43.7
Min (cm)	-70.8	-38.9
Mean(cm)	-29.2	0.0
Std± (cm)	16.5	14.2
RMS (cm)	33.4	13.9

Table 6: Statistical summary using CG2 to derive the coefficients for the similarity transformation CS8 at 26 GPS points, for Perak.

	No CS	CS8
# pts	27	27
Max (cm)	178.4	74.1
Min (cm)	-47.6	-65.4
Mean(cm)	37.4	0.0
Std± (cm)	49.6	30.6
RMS (cm)	61.4	30.0

Table 7: Statistical summary using CG2 to derive the coefficients for the similarity transformation CS8 at 27 GPS points, for Pahang.

	No CS	CS8
# pts	11	11
Max (cm)	154.8	33.5
Min (cm)	-24.7	-22.6
Mean(cm)	44.4	0.0
Std± (cm)	60.3	16.3
RMS (cm)	72.6	15.6

Table 8: Statistical summary using CG2 to derive the coefficients for the similarity transformation CS8 at 11 GPS points, for Negri Sembilan.

	No CS	CS8
# pts	13	13
Max (cm)	66.3	6.7
Min (cm)	-51.6	-15.8
Mean(cm)	9.8	-1.4
Std± (cm)	36.4	7.08
RMS (cm)	36.3	6.9

Table 9: Statistical summary using CG2 to derive the coefficients for the similarity transformation CS8 at 13 GPS points, for Kelantan.

	No CS	CS8
# pts	12	12
Max (cm)	9.0	9.9
Min (cm)	-90.2	-13.2
Mean(cm)	-34.7	0.0
Std± (cm)	25.8	6.0
RMS (cm)	42.6	5.7

Table 10: Statistical summary using CG2 to derive the coefficients for the similarity transformation CS8 at 12 GPS points, for Kedah.

	No CS	CS8
# pts	16	16
Max (cm)	81.3	15.0
Min (cm)	-5.6	-19.3
Mean(cm)	38.2	0.0
Std± (cm)	23.0	9.7
RMS (cm)	44.2	9.4

Table 11: Statistical summary using CG2 to derive the coefficients for the similarity transformation CS8 at 16 GPS points, for Terenganu.

6.0 CONCLUSION

REFERENCES

Cressie, N. A. C., 1991. Statistics for Spatial Data, John Wiley and Sons, Inc., New York, 900 pp.

Fashir, H., Omar, K., Kadir, M., Ses, S., Chen, K.E., Abu, S., and Mohamed, A., 1999. Towards the study of vertical datum inconsistencies in Peninsular Malaysia, *Geomatics Research Australasia,* No. 71, June, 1999, pp. 37-56.

Haagmans, R.R., de Min, E. and van Gelderen, M., 1993. Fast evaluation of convolution integrals on the sphere using 1D-FFT, and a comparison with existing methods for Stokes's integral, *Manuscripta Geodatica*, 18 (5), 227-241.

Heiskanen, W.H. and Moritz, H., 1967. *Physical Geodesy*, W.H. Freeman and Co., San Francisco, USA.

Kadir, Majid., 1983. Investigations of geological influences upon the correlation between the free air gravity anomalies and topography of Finland, Reports of the Finnish Geodetic Institute.

Kadir, A., Majid, A., Fashir, Hassan, H., Omar, Kamaluddin 1999. A Regional Gravimetric Co-Geoid over South East Asia, *Geomatics Research Australasia,* No. 71, June, 1999, pp. 37-56.

Kotsakis, C., and Sideris, M., G., 1999. On the adjustment of combined GPS/Levelling/Geoid networks. *Journal of Geodesy*, (73): 412-421

Mainville, A., Craymer, M., Blackie, S., 1997. The GPS height transformation 1997, an ellipsoidal-orthometric height transformation for use with GPS in Canada. Report of Geodetic Survey Division, Geomatics Canada, Ottawa.

Martensson, S.,G., 2002. Height determination by GPS – Accuracy with respect to different geoid models in Sweden, *FIG XXII International Congress, Washington, D.C. USA, April 19-26 2002.*

Moritz, H., 1980. Advanced Physical Geodesy. H. Wichman Verlag, Karlsruhe.

Schwarz K-P., Sideris, M.G. and Forsberg, R., 1990. The use of the FFT techniques in physical geodesy, *Geophysical Journal International,* 100.

Sideris, M.G. and She, B.B., 1995. A new, high resolution geoid for Canada and part of the US by the 1D-FFT method, *Bulletin Geodesique,* 69(2).

Sideris, M.G., 1999. Geoid determination by FFT techniques. Lecture Notes, International School for the Determination and Use of the Geoid, UTM, University Technology Malaysia, Johor, Malaysia, February.

Zhiheng, J., and Duquenne, H., 1996. On the combined adjustment of a gravimetrically determined geoid and GPS levelling stations, *Journal of Geodesy*, (70): 505-514.

Zhong, D., 1997. Robust estimation and optimal selection of polynomial parameters for the interpolation of GPS geoid heights. *Journal of Geodesy*, (71): 552-561.